Worksheet 1 Solutions

Simplifying Algebraic Expressions

1. Simplify the following expressions.

   (a) $\dfrac{1}{3^{-2}} - \dfrac{1}{3} + \dfrac{1}{4^{-1}}$.

   (b) $\dfrac{(x^2y^{-3})^2}{(y^{-3}x^{-2})^{-2}}$.

   (c) If $f(x) = x^2 + 3x$ and $h \neq 0$, then simplify $\dfrac{f(x+h) - f(x)}{h}$.

   (d) Rationalize $\dfrac{3}{x - \sqrt{x}}$.

Solution.

(a)

$$\frac{1}{3^{-2}} - \frac{1}{3} + \frac{1}{4^{-1}} = 3^2 - \frac{1}{3} + 4^1 = 9 - \frac{1}{3} + 4 = \frac{38}{3}.$$

(b)

$$\frac{(x^2y^{-3})^2}{(y^{-3}x^{-2})^{-2}} = \frac{x^4y^{-6}}{y^6x^4} = \frac{1}{y^{12}}.$$

(c)

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{(x+h)^2 + 3(x+h) - x^2 + 3x}{h} \\ &= \frac{x^2 + 2xh + h^2 + 3x + 3h - x^2 - 3x}{h} \\ &= \frac{2xh + h^2 + 3h}{h} \\ &= 2x + h + 3.\end{aligned}$$

(d)

$$\frac{3}{x - \sqrt{x}} = \frac{3}{x - \sqrt{x}} \left(\frac{x + \sqrt{x}}{x + \sqrt{x}} \right) = \frac{3(x + \sqrt{x})}{x^2 - x}.$$

Intervals

2. Write the following in interval notation.

   (a) The open interval with endpoints 2 and 3.

(b) The half-open interval with endpoints 2 and 3 that contains 2 but not 3.

Solution.
(a) $(2, 3)$
(b) $[2, 3)$

Solving Equations

3. Solve for x: $2y^2x - y^2 - (1 + 3y) = x$.

Solution.
Bring all x terms to left-hand side and y terms to right-hand side:

$$\begin{aligned} 2y^2x - x &= y^2 + (1 + 3y) \\ x(2y^2 - 1) &= y^2 + (1 + 3y) \\ x &= \frac{y^2 + 1 + 3y}{2y^2 - 1}. \end{aligned}$$

4. Find the solutions of $\frac{x^2}{3} + 2x - 1 = 0$ exactly.
Multiply by 3 then use the quadratic formula:

$$\begin{aligned} x^2 + 6x - 3 = 0 & \\ x &= \frac{-6 \pm \sqrt{36 - 4(1)(-3)}}{2(1)} \\ &= -3 \pm \frac{\sqrt{48}}{2} \\ &= -3 \pm 2\sqrt{3}. \end{aligned}$$

5. Find the solutions of $\frac{1}{x-4} + \frac{1}{x+4} = \frac{4}{x^2 - 16}$ exactly.

Solution.
Find common denominator and then solve:

$$\begin{aligned} \frac{1}{x-4} + \frac{1}{x+4} &= \frac{4}{x^2 - 16} \\ \frac{1}{x-4} + \frac{1}{x+4} &= \frac{4}{(x+4)(x-4)} \\ \frac{(x+4) + (x-4)}{(x+4)(x-4)} &= \frac{4}{(x+4)(x-4)} \\ (x+4) + (x-4) &= 4 \\ 2x &= 4 \\ x &= 2 \end{aligned}$$

Exponential and Logarithmic Functions

6. Simplify the following.

(a) $\dfrac{2^{5x}}{2^x}$ (b) $e^{2x}e^{-3x}$

Solution.
(a)

$$\frac{2^{5x}}{2^x} = 2^{5x-x} = 2^{4x}.$$

(b)

$$e^{2x}e^{-3x} = e^{2x+(-3x)} = e^{-x}.$$

7. Evaluate $\log_4(1/64)$.

Solution.

$$\log_4(1/64) \iff 4^x = \frac{1}{64} \iff x = -3.$$

Check: $4^{-3} = \dfrac{1}{4^3} = \dfrac{1}{64}$.

8. Solve for t in the equation $\ln(t) - \ln(t^2) = 5$ exactly.

Solution.
Using properties of logs:

$$\begin{aligned} \ln(t) - \ln(t^2) &= 5 \\ \ln\left(\frac{t}{t^2}\right) &= 5 \\ \ln\left(\frac{1}{t}\right) &= 5 \\ e^{\ln(1/t)} &= e^5 \\ \frac{1}{t} &= e^5 \\ t &= \frac{1}{e^5} \end{aligned}$$

Trigonometric Functions

9. On the unit circle mark off the following angles (in radians):

(a) $\dfrac{\pi}{2}, \pi$, and $-\dfrac{\pi}{2}$ together (b) $\dfrac{\pi}{3}$ and $\dfrac{2\pi}{3}$ together.

Solution.
See trig. review sheet.

Inverse Functions

10. Find the inverse of each of the following functions, including the domain.

(a) $f(x) = \dfrac{x}{1+2x}$ for $x \neq -\dfrac{1}{2}$

(b) $f(x) = \sqrt{18-2x^2}$ for $0 \leq x \leq 3$.

(c) $f(x) = \ln(e^{2x}+1)$ for all x.

Solution.
Steps to find the inverse of a function:

1. Set $y = f(x)$
2. Switch y and x
3. Solve for y

(a) Let $y = f(x) = \dfrac{x}{1+2x}$. Swapping y and x and then solving:

$$\begin{aligned} x &= \frac{y}{1+2y} \\ x(1+2y) &= y \\ x + 2xy &= y \\ 2xy - y &= -x \\ y(2x-1) &= -x \\ y &= -\frac{x}{2x-1}. \end{aligned}$$

So

$$f^{-1}(x) = -\frac{x}{2x-1}.$$

Now the only place that $f^{-1}(x)$ is undefined is when $x = 1/2$. Domain: All real numbers except $x = \dfrac{1}{2}$.

(b) Let $y = f(x) = \sqrt{18-2x^2}$. Since $0 \leq x \leq 3$ we have that

$$0 \leq y \leq \sqrt{18}.$$

Swapping y and x and then solving:

$$\begin{aligned} x &= \sqrt{18-2y^2} \\ x^2 &= 18 - 2y^2 \\ 2y^2 &= 18 - x^2 \\ y &= \pm\sqrt{\frac{18-x^2}{2}}. \end{aligned}$$

But since y is always positive we just take the positive root as our answer:

$$f^{-1}(x) = \sqrt{\frac{18-x^2}{2}}.$$

Domain: $0 \leq x \leq \sqrt{18}$.

(c) Let $y = f(x) = \ln(e^{2x} + 1)$. Swapping x and y and then solving:

$$\begin{aligned} x &= \ln(e^{2y} + 1) \\ e^x &= e^{2y} + 1 \\ e^{2y} &= e^x - 1 \\ 2y &= \ln(e^x - 1) \\ y &= \frac{1}{2}\ln(e^x - 1). \end{aligned}$$

So

$$f^{-1}(x) = \frac{1}{2}\ln(e^x - 1).$$

Now $\ln(x)$ is only defined for $x > 0$. So we must solve for which x-values make $e^x - 1 > 0$.

$$\begin{aligned} e^x - 1 &> 0 \\ e^x &> 1 \\ x &> \ln(1) = 0. \end{aligned}$$

Domain: $x > 0$.