Worksheet 6: (4.4-5.2)

Name: ________________________________ Due: June 27th by 3PM via HuskyCT

Indeterminate Forms and L'Hospital's Rule

1. State each limit's indeterminate form and then compute the limit. If l'Hospital's Rule is needed more than once, try to simplify the expression before applying it.

(a) $\lim_{x \to 0} \frac{(x+1)^9 - 9x - 1}{x^2}$

(b) $\lim_{x \to 0} \frac{\sin(3x)}{e^{9x} - e^{2x}}$

(c) $\lim_{x \to 0^+} \frac{\ln(\sin(2x))}{\ln(\sin(3x))}$

(d) $\lim_{x \to 0} \frac{\ln(\cos(2x))}{\ln(\cos(3x))}$

(e) $\lim_{x \to 1^+} \left(\frac{1}{\ln(x)} - \frac{1}{x-1} \right)$

Optimization

2. Find the dimensions of a rectangle with area 512 m^2 whose perimeter is as small as possible.

3. A farmer with 750 ft of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the four pens?

4. Find the point on the line $y = 5x + 2$ that is closest to the origin.

Antiderivatives and Definite Integrals

5. Find the most general antiderivatives of the following functions (use C as any constant).

(a) $f(x) = \frac{1}{2} + \frac{3}{4}x^2 - \frac{4}{5}x^3$

(b) $f(t) = \frac{t^4 + 3\sqrt{t}}{t^2}$

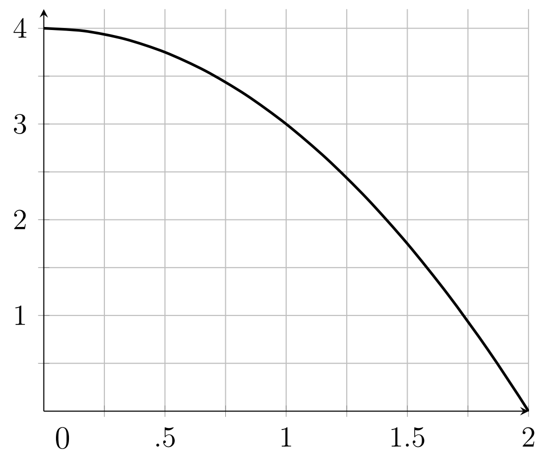
(c) $g(\theta) = \cos\theta - 5\sin\theta$

6. Find a function $f(x)$ satisfying the following conditions:

(a) $f'''(x) = \cos(x),\ f(0) = 1,\ f'(0) = 2,\ f''(0) = 3$

(b) $f''(x) = 2 - 12x,\ f(0) = 9,\ f(2) = 7$

7. The graph of $y = 4 - x^2$ over the interval $[0, 2]$ is given below.



(a) Estimate the area under the graph over $[0, 2]$ using 4 rectangles and right-endpoints. Sketch the rectangles in the graph above.

(b) Estimate the area under the graph over $[0, 2]$ using 4 rectangles and left-endpoints.

(c) Estimate the area under the graph over $[0, 2]$ using 4 rectangles and midpoints.

(d) Calculate the exact area under $y = 4 - x^2$ over $[0, 2]$ by going to Wolframalpha.com and typing in "integral from 0..2 of 4-x^2". Which of the approximations in (a), (b), and (c) is closest to this?

Extra Credit: If an object is dropped from rest, one model for its speed v after t seconds, taking air resistance into account, is

$$v = \frac{mg}{c}(1 - e^{-ct/m}),$$

where g is the acceleration due to gravity and c is a positive constant describing air resistance.
(a) Calculate $\lim_{t\to\infty} v$. (*You do not need L'Hospital's Rule to evaluate this limit.*) What is the meaning of this limit?
(b) For fixed t, use L'Hospital's Rule to calculate $\lim_{c\to 0^+} v$.